Mathematics 664 - Fall 2021
Mathematical Theory of the Navier-Stokes Equations

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Title: Mathematical Theory of the Navier-Stokes Equations.

Time: TR 9:35am-10:50am

Course description: In these lectures I will introduce the basics of the mathematical theory of the Navier-Stokes equations of viscous fluids. These equations appear in a wide range of physical and biological applications, varying from oceanic and atmospheric dynamics, to combustion theory any body-fluid transport. From the mathematical point of view, these equations have been identified among the most challenging problems in Applied Analysis. On the other hand, from the computational point of view they are prohibitively expensive to simulate, and out of reach even for most powerful state-of-the-art computers.

In this course we will mainly focus on the analytical properties of the solutions to the **incompressible** Navier-Stokes equations. Students who are interested in attending the course are expected to have a background in Real Analysis (and some basics of Functional Analysis). The rest of the course will be, to a large extend, self-contained.

Proposed topics to be covered:

1. Deriving the Navier-Stokes and Euler equations from basic physical principles. Introducing the appropriate Functional Spaces.
2. Steady state solutions to the incompressible Navier-Stokes equations and their regularity.
3. Time dependent Leray-Hopf weak solutions to the incompressible Navier-Stokes equations.
4. Global regularity of strong solutions for the two-dimensional incompressible Navier-Stokes equations
5. Short time existence of strong solutions in the three-dimensional incompressible Navier-Stokes equations. case.
6. Weak-strong uniqueness and the role of the energy inequality in the Leray-Hopf weak solutions.

7. Time Analyticity and Gevrey regularity of strong solutions to incompressible Navier-Stokes equations
8. Long-time behavior of solutions, global attractors, determining modes, nodes and other degrees of freedom (if time allows)
9. Inertial Manifolds (if time allows).

Textbooks:

1. P. Constantin and C. Foias, Navier-Stokes Equations, University of Chicago Press, 1988.
2. R. Temam, Navier-Stokes Equations: Theory and Numerical Analysis, North-Holand. New print published by the AMS 2001.
3. R. Temam, Navier-Stokes Equations and Nonlinear Functional Analysis, CBMS-NSF Regional Conference Series in Applied Math 66, SIAM, 2nd Ed, 1995.
4. R. Temam, Infinite Dimensional Dynamical Systems in Mechanics and Physics, 2nd Ed, Applied Math Sci. 68, Springer-Verlag, 1997.
5. H. Sohr, The Navier–Stokes Equations, An Elementary Functional Analytic Approach, Birkhäuser Verlag, Basel, 2001.

Other References:

1. A. Chorin and J. Marsden, A Mathematical Introduction to Fluid Mechanics, Springer-Verlag.
2. C. Doering and J. Gibbon, Applied Analysis of the Navier-Stokes Equations, Cambridge University Press.
3. C. Foias, O. Manley, R. Rosa and R. Temam, Navier-Stokes Equations and Turbulence, Cambridge University Press, Cambridge, 2001.
4. A. Majda and A. Bertozzi, Vorticity and Incompressible Flow, Cambridge University Press, 2002.
5. J. C. Robinson, Infinite-dimensional Dynamical Systems: An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors, Cambridge Texts in Applied Mathematics.