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# SYLLABUS

## SPECIAL TOPICS IN REPRESENTATION THEORY AND INTEGRABLE PROBABILITY MATH 689 FALL 2020

TBD, Blocker TBD  
Office Hours: TBD

### Course description

This class will cover the representation theory background necessary for an introduction to integrable probability. Some topics will be:

- (1) Basics of continuous-time Markov chains on discrete spaces.
  - (2) Schur's lemma, the Peter-Weyl theorem, the parametrization of finite-dimensional irreducible representations of the compact Lie group  $SU(n)$  as Young diagrams. Applications to Schur processes will be discussed.
  - (3) The construction and representations of the Drinfeld-Jimbo quantum group  $U_q(\mathfrak{sl}_2)$ , with applications to the asymmetric simple exclusion process.
- Time permitting, topic (1) can be lifted to the connection of Macdonald polynomials to double affine Hecke algebras, with applications to Macdonald processes; topic (2) can be lifted to quantized affine Lie algebras and applications to vertex models.

### Prerequisites

Familiarity with groups and rings (such as Math 415). Background in measure theory or probability theory is beneficial, but not required.

### Learning Outcomes:

Students should aim to understand the connection between representation theory and integrable probability.

### Homework

There will be a weekly homework assignment, due in class on TBD.

### Exams,Labs

There are no exams or labs.

### Grading Policy

Attendance will count for 80% and homework will count for 20%. The grades will be distributed as 90 – 100% for an A, 80 – 89% for a B, 70 – 79% for a C, and 60 – 69% for a D, and 0 – 59% for a F.

### Weekly Schedule

1. Simple random walks and Fourier Series
2. Basics of Probability
3. Basics of Markov chains
4. Schur's Lemma and Characters
5. Semisimple Lie Algebras
6. Engel's and Lie's Theorem

7. Representations of  $\mathfrak{sl}_2$
8. Classification of Representations of  $\mathfrak{sl}_n$
9. Symmetric polynomials as characters; Schur processes
10. Symmetric Exclusion Processes and Hopf algebras
11. Drinfeld–Jimbo quantum groups
12. Spin Chains
13. ASEP duality
14. Stochastic vertex models

**Attendance and Make-up Policies:**

Please read: See <http://student-rules.tamu.edu/rule07>.

**Americans with Disabilities Act (ADA):**

The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, please contact Disability Services, currently located in the Disability Services building at the Student Services at White Creek complex on west campus or call 979-845-1637. For additional information, visit <http://disability.tamu.edu>.

**Academic Integrity:**

For additional information please visit: <http://aggiehonor.tamu.edu>

”An Aggie does not lie, cheat, or steal, or tolerate those who do.”

**Title IX and Statement on Limits to Confidentiality**

Texas A&M University and the College of Science are committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws provide guidance for achieving such an environment. Although class materials are generally considered confidential pursuant to student record policies and laws, University employees including instructors cannot maintain confidentiality when it conflicts with their responsibility to report certain issues that jeopardize the health and safety of our community. As the instructor, I must report (per Texas A&M System Regulation 08.01.01) the following information to other University offices if you share it with me, even if you do not want the disclosed information to be shared:

Allegations of sexual assault, sexual discrimination, or sexual harassment when they involve TAMU students, faculty, or staff, or third parties visiting campus.

These reports may trigger contact from a campus official who will want to talk with you about the incident that you have shared. In many cases, it will be your decision whether or not you wish to speak with that individual. If you would like to talk about these events in a more confidential setting, you are encouraged to make an appointment with the Student Counseling Service (<https://scs.tamu.edu/>).

Students and faculty can report non-emergency behavior that causes them to be concerned at <http://tellsomebody.tamu.edu>